



## Dictionary of Mong Njua and the Graphical Law

\*<sup>1</sup>Anindya Kumar Biswas

<sup>1</sup>Department of Physics, North-Eastern Hill University, Mawkyntshing, Shillong-793022, Meghalaya, India.

### Abstract

We study the head entries of Dictionary of Mong Njua by Thomas Amis Lyman. We draw the natural logarithm of the number of head entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We conclude that the Dictionary can be characterised by BP(4,  $\beta H=0$ ), i.e. the magnetisation curve in the Bethe-Peierls approximation of the Ising Model, in the presence of four nearest neighbours and in the absence of external magnetic field,  $H$ , with  $\beta H=0$ .  $\beta$  is  $\frac{1}{k_B T}$  where,  $T$  is temperature and  $k_B$  is the tiny Boltzmann constant.

**Keywords:** Mong Njua, Dictionary, Ranking, Magnetisation, Ising Model, Bethe-Peierls approximation.

### Introduction

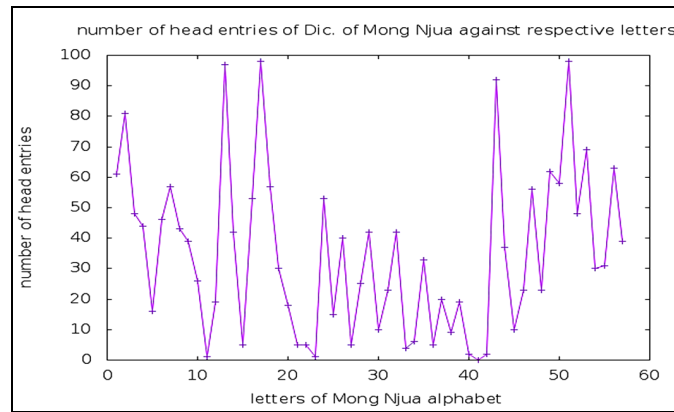
"The Mong Njua tribe is also known as 'Green Miao' or 'Green Meo'. The tribe itself, however, uses the ethnic name Mong to which a descriptive term is added to designate the particular branch of the Mong group. In reference to themselves, the tribesmen used the designator nju'a, 'to be green, be azure'. The Mong constitute a sub-division of the ethnic group known as Miao who inhabits the mountainous regions of Southwestern China, North Vietnam, Laos and the Shan states of Burma and northern Thailand. The Miao are divided by linguistic and cultural differences into a number of

tribes whose names often derive from the color or pattern of their women's garments (White Miao, Black Miao, Striped Miao etc). The Mong Njua or 'Green Miao' with 'White Miao' and 'Banded-Sleeve Miao' are the three sub-groups of Miao to be found in Thailand.....".....Dictionary of Mong Njua, <sup>[1]</sup>.

In this paper, we turn to Dictionary of Mong Njua, <sup>[1]</sup>. We go through the head entries. We count all the head entries of the dictionary, <sup>[1]</sup>, one by one from the beginning to the end. The result is the table, Table 1. To visualise we plot the number of head entries against the respective letters in the dictionary sequence, <sup>[1]</sup>, in the adjoining figure, Figure1.

**Table 1:** The head entries of Dictionary of Mong Njua against Mong Njua alphabet, <sup>[1]</sup>.

?	c	ch	č	čh	ć	ćh	f	h	hl	hs
61	81	48	44	16	46	57	43	39	26	1
hy	k	kh	khl	kl	l	m	mb	mbh	ml	mph
19	97	42	5	53	98	57	30	18	5	5
mphl	n	n ċh	nd	nth	nz	ñ	ñch	ñdj	ñj	ñt <sub>h</sub>
1	53	15	40	5	25	42	10	23	42	4
ñčh	ñj	ng	ng g	ng gl	ngy	ng kh	ng khl	ng qh	p	ph
6	33	5	20	9	19	2	0	2	92	37
p <sub>h</sub> l	pl	q	qh	s	š	t	th	t <sub>h</sub>	t <sub>h</sub> h	v
10	23	56	23	62	58	98	48	69	30	31
									y	ž
									63	39



**Fig 1:** The vertical axis is the number of the head entries of Dictionary of Mong Njua <sup>[1]</sup>. The horizontal axis is the letters of the Mong alphabet. Letters are represented by the sequence number in the alphabet as it appears in the dictionary, <sup>[1]</sup>.

Next we look for the graphical law. We have started considering magnetic field pattern in <sup>[2]</sup>, in the languages we converse with. We have studied there, a set of natural languages, <sup>[2]</sup>, and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate, <sup>[3]</sup>, into dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in the references from <sup>[4]</sup> to <sup>[103]</sup>.

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of the head entries of Dictionary of Mong Njua, <sup>[1]</sup>. In the section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

### The Graphical Law Analysis

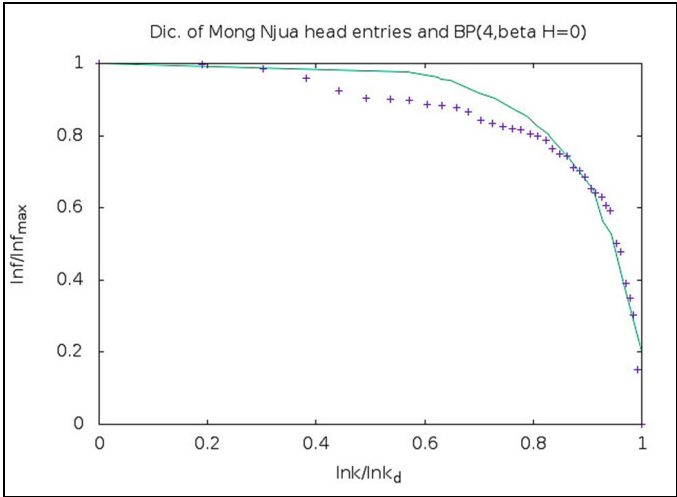
For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. The lowest value of  $f$  is one. The corresponding rank,  $k$ , denoted as  $k_{\text{lim}}$  is thirty eight. As a result both  $\frac{\ln f}{\ln f_{\text{max}}}$  and  $\frac{\ln k}{\ln k_{\text{lim}}}$  varies from zero to one. Then we tabulate in the adjoining table, Table-2, and plot  $\frac{\ln f}{\ln f_{\text{max}}}$  against  $\frac{\ln k}{\ln k_{\text{lim}}}$  in the figure, Figure 2.

We then ignore the letter with the highest number of head entries starting with, tabulate in the adjoining table, Table-2, and redo the plot, normalising the  $\ln f$  s with next-to-maximum  $\ln f_{n-\text{max}}$ , and starting from  $k=2$  in the figure, Figure 3. This program then we repeat up to  $k=5$ , resulting in the figures up to Figure 7.

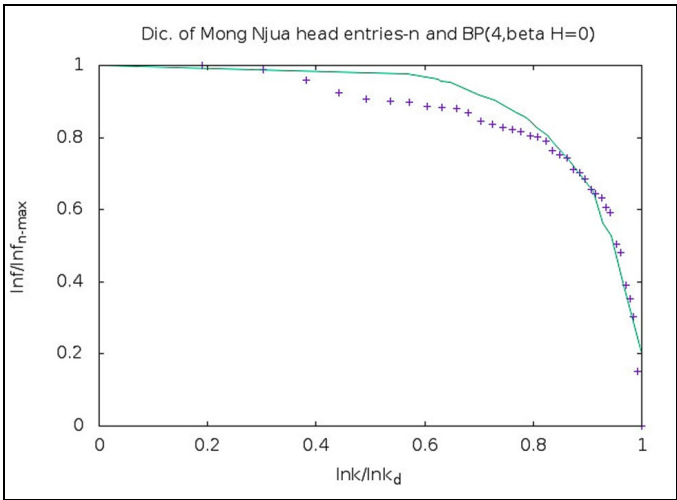
**Table 2:** The head entries of Dictionary of Mong Njua: ranking, natural logarithm, normalisations

k	lnk	$\frac{\ln k}{\ln k_{\text{lim}}}$	f	lnf	$\frac{\ln f}{\ln f_{\text{max}}}$	$\frac{\ln f}{\ln f_{n\text{max}}}$	$\frac{\ln f}{\ln f_{2n\text{max}}}$	$\frac{\ln f}{\ln f_{3n\text{max}}}$	$\frac{\ln f}{\ln f_{4n\text{max}}}$	$\frac{\ln f}{\ln f_{5n\text{max}}}$
1	0	0	98	4.585	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.190	97	4.575	0.998	1	Blank	Blank	Blank	Blank
3	1.10	0.302	92	4.522	0.986	0.988	1	Blank	Blank	Blank
4	1.39	0.382	81	4.394	0.958	0.960	0.972	1	Blank	Blank
5	1.61	0.442	69	4.234	0.923	0.925	0.936	0.964	1	Blank
6	1.79	0.492	63	4.143	0.904	0.906	0.916	0.943	0.979	1
7	1.95	0.536	62	4.127	0.900	0.902	0.913	0.939	0.975	0.996
8	2.08	0.571	61	4.111	0.897	0.899	0.909	0.936	0.971	0.992
9	2.20	0.604	58	4.060	0.885	0.887	0.898	0.924	0.959	0.980
10	2.30	0.632	57	4.043	0.882	0.884	0.894	0.920	0.955	0.976
11	2.40	0.659	56	4.025	0.878	0.880	0.890	0.916	0.951	0.972
12	2.48	0.681	53	3.970	0.866	0.868	0.878	0.904	0.938	0.958
13	2.56	0.703	48	3.871	0.844	0.846	0.856	0.881	0.914	0.934
14	2.64	0.725	46	3.829	0.835	0.837	0.847	0.871	0.904	0.924
15	2.71	0.745	44	3.784	0.825	0.827	0.837	0.861	0.894	0.913
16	2.77	0.761	43	3.761	0.820	0.822	0.832	0.856	0.888	0.908
17	2.83	0.777	42	3.738	0.815	0.817	0.827	0.851	0.883	0.902
18	2.89	0.794	40	3.689	0.805	0.806	0.816	0.840	0.871	0.890
19	2.94	0.808	39	3.664	0.799	0.801	0.810	0.834	0.865	0.884
20	3.00	0.824	37	3.611	0.788	0.789	0.799	0.822	0.853	0.872

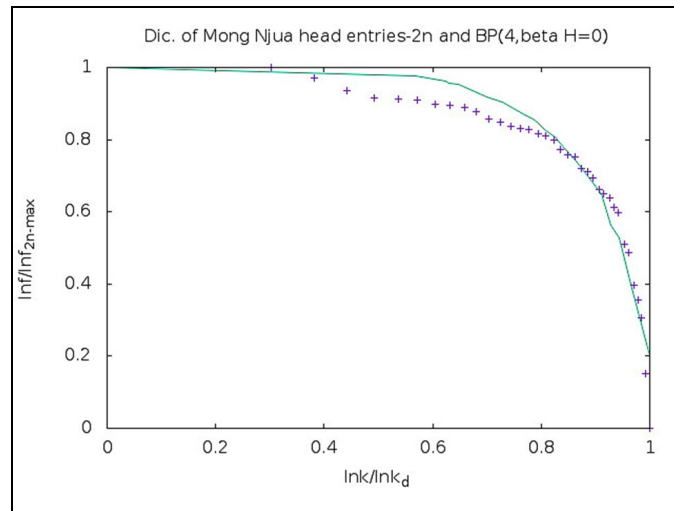
21	3.04	0.835	33	3.497	0.763	0.764	0.773	0.796	0.826	0.844
22	3.09	0.849	31	3.434	0.749	0.751	0.759	0.782	0.811	0.829
23	3.14	0.863	30	3.401	0.742	0.743	0.752	0.774	0.803	0.821
24	3.18	0.874	26	3.258	0.711	0.712	0.720	0.741	0.769	0.786
25	3.22	0.885	25	3.219	0.702	0.704	0.712	0.733	0.760	0.777
26	3.26	0.896	23	3.135	0.684	0.685	0.693	0.713	0.740	0.757
27	3.30	0.907	20	2.996	0.653	0.655	0.663	0.682	0.708	0.723
28	3.33	0.915	19	2.944	0.642	0.643	0.651	0.670	0.695	0.711
29	3.37	0.926	18	2.890	0.630	0.632	0.639	0.658	0.683	0.698
30	3.40	0.934	16	2.773	0.605	0.606	0.613	0.631	0.655	0.669
31	3.43	0.942	15	2.708	0.591	0.592	0.599	0.616	0.640	0.654
32	3.47	0.953	10	2.303	0.502	0.503	0.509	0.524	0.544	0.556
33	3.50	0.962	9	2.197	0.479	0.480	0.486	0.5	0.519	0.530
34	3.53	0.970	6	1.792	0.391	0.392	0.396	0.408	0.423	0.433
35	3.56	0.978	5	1.609	0.351	0.352	0.356	0.366	0.380	0.388
36	3.58	0.984	4	1.386	0.302	0.303	0.307	0.315	0.327	0.335
37	3.61	0.992	2	0.693	0.151	0.151	0.153	0.158	0.164	0.167
38	3.64	1	1	0	0	0	0	0	0	0



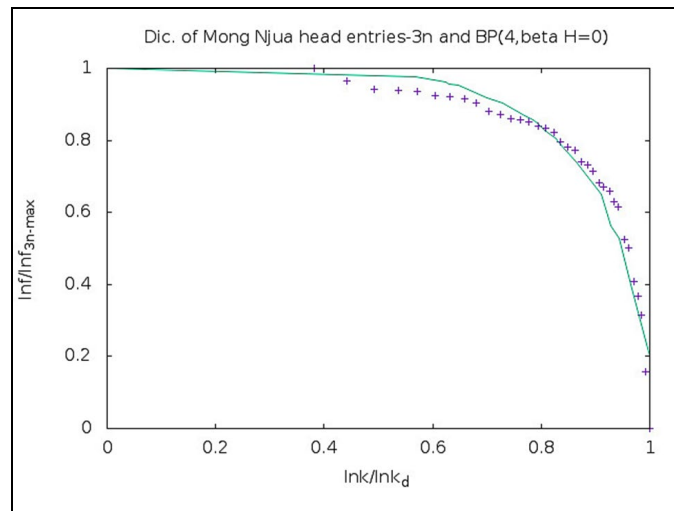
**Fig 2:** The vertical axis is  $\frac{\ln f}{\ln f_{\max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the entries of Dictionary of Mong Njua, with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H=0$ ), of the Ising Model, in the presence of four nearest neighbours and in the absence of external magnetic field,  $H = 0$ .



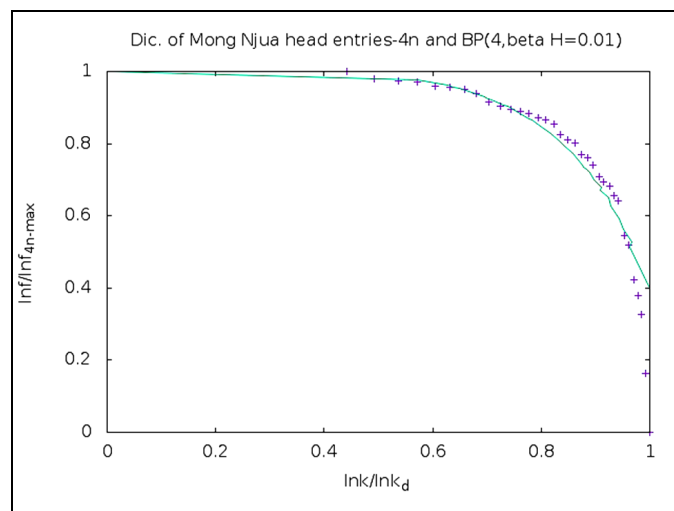
**Fig 3:** The vertical axis is  $\frac{\ln f}{\ln f_{n\max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the entries of Dictionary of Mong Njua, with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H=0$ ), of the Ising Model, in the presence of four nearest neighbours and in the absence of external magnetic field,  $H = 0$ .



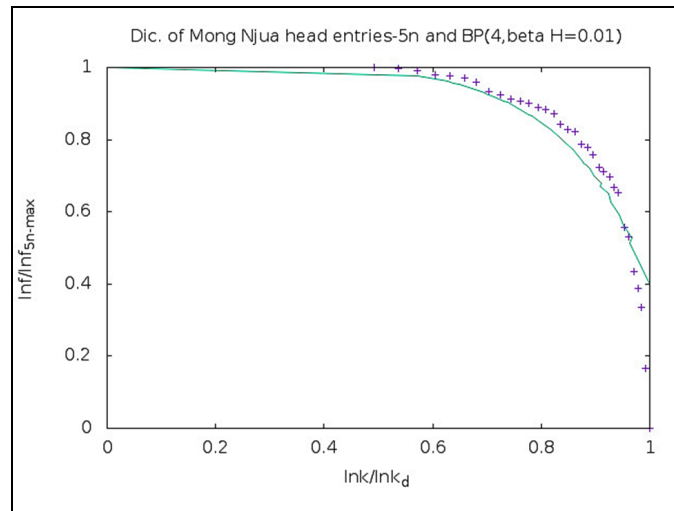
**Fig 4:** The vertical axis is  $\frac{\ln f}{\ln f_{2n,\max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the entries of Dictionary of Mong Njua, with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H=0$ ), of the Ising Model, in the presence of four nearest neighbours and in the absence of external magnetic field,  $H=0$ .



**Fig 5:** The vertical axis is  $\frac{\ln f}{\ln f_{3n,\max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the entries of Dictionary of Mong Njua, with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H=0$ ), of the Ising Model, in the presence of four nearest neighbours and in the absence of external magnetic field,  $H=0$ .



**Fig 6:** The vertical axis is  $\frac{\ln f}{\ln f_{4n,\max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the entries of Dictionary of Mong Njua, with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H=0.01$ ), of the Ising Model, in the presence of four nearest neighbours and in the presence of external magnetic field,  $H$ , with  $\beta H=0.01$ .



**Fig 7:** The vertical axis is  $\frac{\ln f}{\ln f_{\max}}$  and the horizontal axis is  $\frac{\ln k}{\ln k_d}$ . The + points represent the entries of Dictionary of Mong Njua, with the fit curve being the Bethe-Peierls curve, BP(4,  $\beta H=0.01$ ), of the Ising Model, in the presence of four nearest neighbours and in the presence of external magnetic field, H, with  $\beta H=0.01$ .

## Conclusion

From the figures, Figure 2-Figure 7, we observe that there is a curve of magnetisation, behind the entries of Dictionary of Mong Njua <sup>[1]</sup>. This is the magnetisation curve BP(4,  $\beta H=0$ ), in the Bethe-Peierls approximation of the Ising Model, in the presence of four nearest neighbours and in the absence of external magnetic field, H, with  $\beta H=0$ .

Moreover, the associated correspondence is,

$$\frac{\ln f}{\ln f_{\max}} \leftrightarrow \frac{M}{M_{\max}} \text{ and } \ln k \leftrightarrow T,$$

k corresponds to temperature in an exponential scale, <sup>[11]</sup>.

## Magnetisation

### Bragg-Williams approximation:

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like paramagnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins,

two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter.

Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by  $L = 1/N \sum_i \sigma_i$  where,  $\sigma_i$  is i-th spin, N being total number of spins. L can vary from minus one to one.  $N = N_+ + N_-$ , where,  $N_+$  is the number of up spins,  $N_-$  is the number of down spins.  $L = \frac{1}{N} (N_+ - N_-)$ . As a result,  $N_+ = \frac{N}{2} (1 + L)$  and  $N_- = \frac{N}{2} (1 - L)$ . Magnetisation or, net magnetic moment, M is  $\mu \sum_i \sigma_i$  or,  $\mu(N_+ - N_-)$  or,  $\mu N L$ ,  $M_{\max} = \mu N$ .  $\frac{M}{M_{\max}} = L$ .  $\frac{M}{M_{\max}}$  is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian, <sup>[105]</sup>, for the lattice of spins, setting  $\mu$  to one, is  $-\epsilon \sum_{n,n} \sigma_i \sigma_j - H \sum_i \sigma_i$ , where n.n refers to nearest neighbour pairs. The difference of energy,  $\Delta E$ , if we flip an up spin to down spin is, <sup>[106]</sup>,  $2\epsilon\gamma\bar{\sigma} + 2H$ , where  $\gamma$  is the number of nearest neighbours of a spin. According to Boltzmann principle  $\frac{N_-}{N_+}$  equals  $\exp(-\Delta E/k_B T)$ , <sup>[107]</sup>. In the Bragg-Williams approximation, <sup>[108]</sup>,  $\bar{\sigma}=L$ , considered in the thermal average sense. Consequently,

$$\ln \frac{1+L}{1-L} = 2 (\gamma\epsilon L + H) / (k_B T) = 2 (L + H/\gamma\epsilon) / (T/(\gamma\epsilon/k_B)) \\ = 2(L+c)/(T/T_c) \quad (1)$$

Where,  $c = H/\epsilon\gamma$ .  $T_c = \epsilon\gamma/k_B$ , <sup>[109]</sup>.  $T/T_c$  is referred to as reduced temperature. Plot of L vs  $T/T_c$  or, reduced

magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field,  $c \neq 0$ , the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice.

To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to Figure 12.12 of the book, [106]. W. L. Bragg was a professor of Hans Bethe. Rudlof Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudlof Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

### Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field:

In the approximation scheme which is improvement over the Bragg-Williams, [105], [106], [107], [108], [109], due to Bethe-Peierls,

[110], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in absence of external magnetic field.

$$(\ln \gamma/(\gamma-2))/(\ln (\text{factor}-1)/D) = T/T_c \quad (2)$$

Where,  $D = \frac{\text{factor}^{(\gamma-1)/\gamma} - \text{factor}^{(1/\gamma)}}{\text{factor}^{(1/\gamma)} - 1}$  and  $\text{factor} = \frac{(1 + \frac{M}{M_{\max}})}{(1 - \frac{M}{M_{\max}})}$ . For  $\gamma=4$  i.e. for four nearest neighbours,  $\ln \frac{\gamma}{\gamma-2}$  is 0.693.

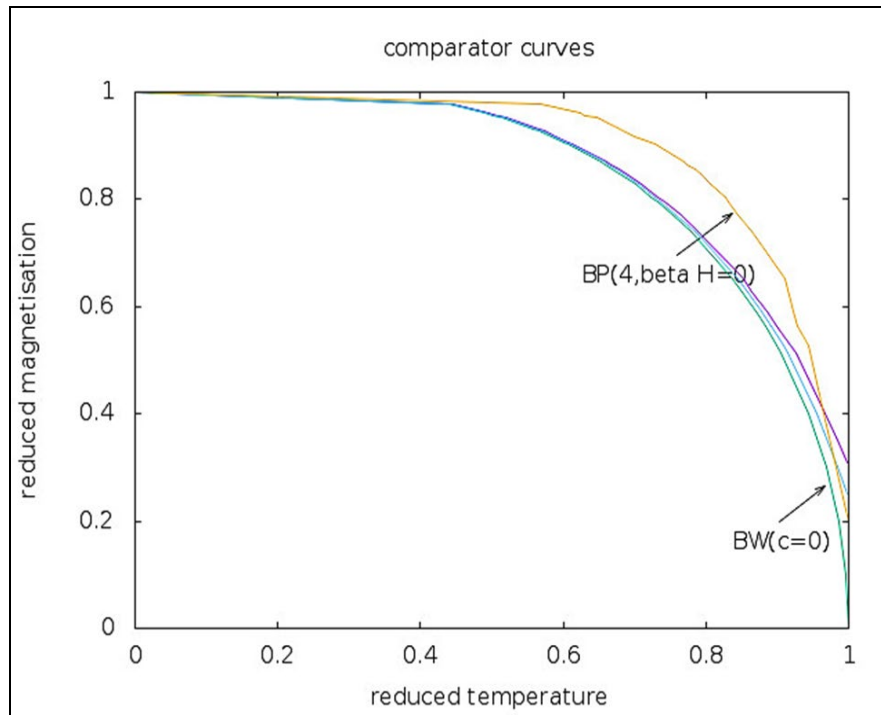
In the following, we describe datas generated from the Equation 1 and the Equation 2 in the table, Table-3, and curves of magnetisation plotted on the basis of those datas. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the Equation 1. BP(4,  $\beta H=0$ ) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the Equation 2. The data set is used to plot Figure 8. Empty spaces in the table, Table-3, mean corresponding point pairs were not used for plotting a line.

**Table 3:** Datas for Reduced temperature [for the Bragg-Williams approximation, in the absence (BW( $c=0$ )) and in the presence (BW( $c=0.005$ ), BW( $c=0.01$ )) of magnetic field,  $c=0$ ,  $c=\frac{H}{T_c}=0.005$ ,  $c=\frac{H}{T_c}=0.01$  respectively and in the Bethe-Peierls approximation, BP(4,  $\beta H=0$ ), in the absence of magnetic field, for four nearest neighbours] vs reduced magnetisation. Reduced temperature data set (say, data set BW( $c=0$ )) is drawn along the x-axis and Reduced magnetisation data set is drawn along the y-axis. In gnuplot the command is plot ".dat" using 1:2 with line; 1 standing for x-axis and 2 standing for y-axis datas.

Reduced temperature, $\frac{T}{T_c}$				$\frac{M}{M_{\max}}$
BW( $c=0$ )	BW( $c=0.005$ )	BW( $c=0.01$ )	BP(4, $\beta H=0$ )	Reduced magnetisation
0	0	0	0	1
0.435	0.437	0.439	0.563	0.978
0.439	0.441	0.443	0.568	0.977
0.491	0.493	0.495	0.624	0.961
0.501	0.504	0.507	0.630	0.957
0.514	0.517	0.519	0.648	0.952
0.559	0.562	0.565	0.654	0.931
0.566	0.569	0.573	0.7	0.927
0.584	0.587	0.590	0.7	0.917
0.601	0.604	0.607	0.722	0.907
0.607	0.610	0.613	0.729	0.903
0.653	0.658	0.661	0.770	0.869
0.659	0.663	0.666	0.773	0.865
0.669	0.674	0.678	0.784	0.856
0.679	0.684	0.688	0.792	0.847
0.701	0.705	0.709	0.807	0.828
0.723	0.728	0.732	0.828	0.805
0.732	0.736	0.743	0.832	0.796
0.753	0.758	0.766	0.845	0.772
0.779	0.784	0.788	0.864	0.740
0.838	0.844	0.853	0.911	0.651
0.850	0.858	0.864	0.911	0.628
0.870	0.877	0.885	0.923	0.592
0.883	0.891	0.899	0.928	0.564
0.899	0.908	0.918		0.527



0.905	0.914	0.926	0.941	0.513
0.944	0.956	0.968	0.965	0.400
		0.985		0.350
		0.998		0.310
0.969	0.985		0.965	0.300
	0.998			0.250
0.987			1	0.200
0.997			1	0.100
1			1	0



**Fig 8:** Reduced magnetisation vs reduced temperature curves, for the Bragg-Williams approximation, in the absence (BW(c=0)) and in the presence (BW(c=0.005), BW(c=0.01)) of magnetic field,  $c=\frac{H}{\gamma\epsilon}=0$ ,  $c=\frac{H}{\gamma\epsilon}=0.005$ ,  $c=\frac{H}{\gamma\epsilon}=0.01$ , outwards; and in the Bethe-Peierls approximation, BP(4,  $\beta H=0$ ), in the absence of magnetic field, for four nearest neighbours (outer in the top).

#### Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field:

In the Bethe-Peierls approximation scheme, <sup>[110]</sup>, reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in presence of external magnetic field, as

$$(\ln \gamma / (\gamma - 2)) / (\ln (\text{factor} - 1) / D) = T / T_c \quad (3)$$

Where,  $D = e^{\frac{2\beta H}{\gamma}} \text{factor}^{((\gamma-1)/\gamma)} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{(1/\gamma)}$  and  $\text{factor} = (1 + \frac{M}{M_{\max}}) / (1 - \frac{M}{M_{\max}})$ . For  $\gamma=4$  i.e. for four nearest neighbours,  $\ln \frac{\gamma}{\gamma-2}$  is 0.693. Derivation of this formula ala <sup>[110]</sup> is given in the appendix of <sup>[7]</sup>. For  $\gamma=4$  i.e. for four nearest neighbours,  $\ln \frac{\gamma}{\gamma-2}$  is 0.693.

$$\text{For four neighbours, } 0.693 / (\ln (\text{factor} - 1) / D) = T / T_c \quad (4)$$

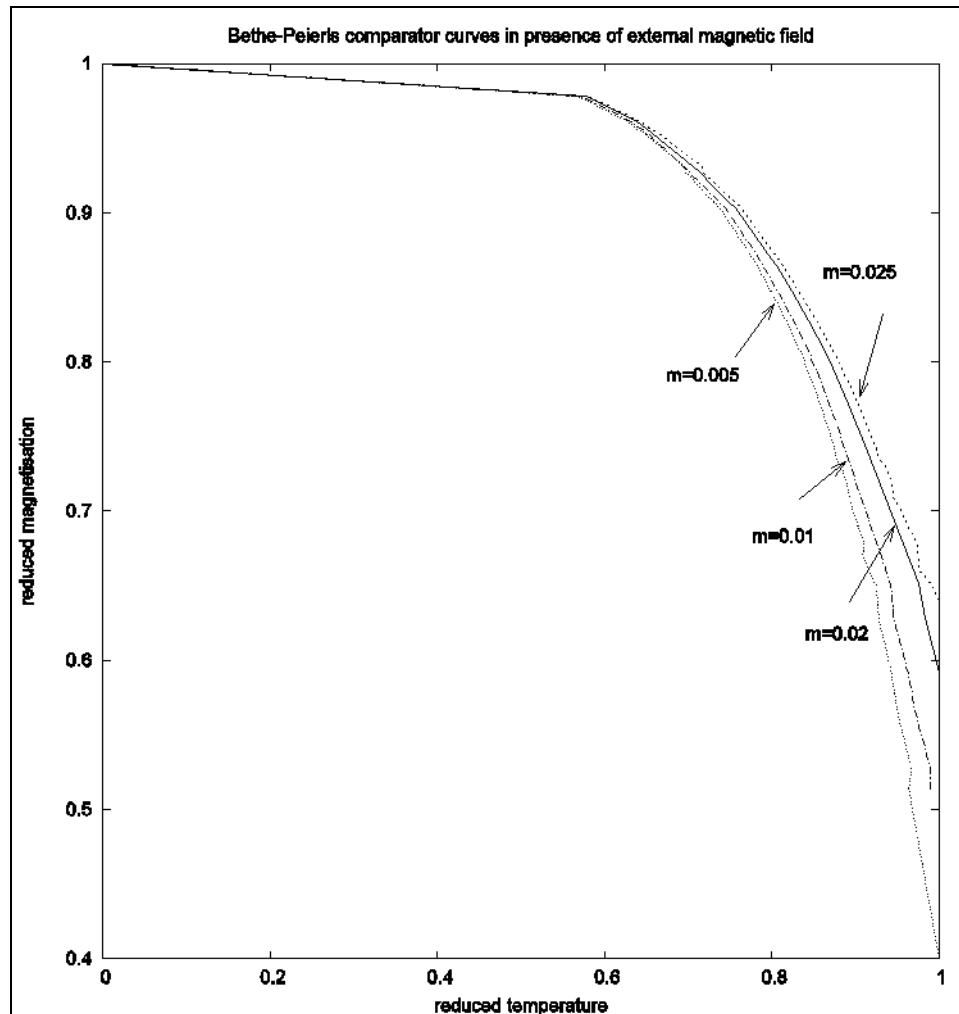
In the following, we describe datas in the table, Table-4, generated from the Equation 4 and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for

reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.06$  calculated from the Equation 4. BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.05$  calculated from the Equation 4. BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.04$  calculated from the Equation 4. BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.02$  calculated from the Equation 4. BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.01$  calculated from the Equation 4. The data set is used to plot Figure 9. Empty spaces in the table, Table-4, mean corresponding point pairs were not used for plotting a line.

Table 4: Bethe-Peierls approx. in presence of little external magnetic fields

Reduced Temperature, $\frac{T}{T_c}$					$\frac{M}{M_{max}}$
BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	Reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0





**Fig 9:** Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with  $\beta H = 2m$ .

### Acknowledgment

The author would like to thank the National Library (Government of India), Kolkata, to allow us to use Dictionary of Mong Njua, <sup>[1]</sup>, in its premises. We have used gnuplot for plotting the figures in this paper.

### References

1. Dictionary of Mong Njua, A Miao (meo) language of South East Asia, by Thomas Amis Lyman, University of Copenhagen, 1974; Mouton, The Hague, Paris; © Copyright 1974 in The Netherlands, Mouton and Co. N.V., Publishers, The Hague; Library of Congress Catalog Card No. 72-94484; Printed in Hungary.
2. Biswas Anindya Kumar, Graphical Law beneath each written natural language, arXiv:1307.6235v3[physics.gen-ph]. A preliminary study of words of dictionaries of twenty six languages, more accurate study of words of dictionary of Chinese usage and all parts of speech of dictionary of Lakher(Mara) language and of verbs, adverbs and adjectives of dictionaries of six languages are included.
3. Biswas Anindya Kumar, A discipline of knowledge and the graphical law, *IJARPS* Volume 1(4), p 21, 2014; viXra: 1908:0090[Linguistics].
4. Biswas Anindya Kumar, Bengali language and Graphical law, viXra: 1908:0090[Linguistics].
5. Biswas Anindya Kumar, Basque language and the Graphical Law, viXra: 1908:0414[Linguistics].
6. Biswas Anindya Kumar, Romanian language, the Graphical Law and More, viXra: 1909:0071[Linguistics].
7. Biswas Anindya Kumar, Discipline of knowledge and the graphical law, part II, viXra:1912.0243 [Condensed Matter], International Journal of Arts Humanities and Social Sciences Studies, Volume 5 Issue 2 | February 2020.
8. Biswas Anindya Kumar, Onsager Core of Abor-Miri and Mising Languages, viXra: 2003.0343[Condensed Matter].
9. Biswas Anindya Kumar, Bengali language, Romanisation and Onsager Core, viXra: 2003.0563[Linguistics].
10. Biswas Anindya Kumar, Little Oxford English Dictionary and the Graphical Law, viXra: 2008.0041[Linguistics].
11. Biswas Anindya Kumar, Oxford Dictionary of Social Work and Social Care and the Graphical law, viXra: 2008.0077[Condensed Matter].
12. Biswas Anindya Kumar, Visayan-English Dictionary and the Graphical law, viXra: 2009.0014[Linguistics].

13. Biswas Anindya Kumar, Garo to English School Dictionary and the Graphical law, viXra: 2009.0056[Condensed Matter].
14. Biswas Anindya Kumar, Mursi-English-Amharic Dictionary and the Graphical law, viXra: 2009.0100[Linguistics].
15. Biswas Anindya Kumar, Names of Minor Planets and the Graphical law, viXra: 2009.0158[History and Philosophy of Physics].
16. Biswas Anindya Kumar, A Dictionary of Tibetan and English and the Graphical law, viXra: 2010.0237[Condensed Matter].
17. Biswas Anindya Kumar, Khasi English Dictionary and the Graphical law, viXra: 2011.0011[Linguistics].
18. Biswas Anindya Kumar, Turkmen-English Dictionary and the Graphical law, viXra: 2011.0069[Linguistics].
19. Biswas Anindya Kumar, Webster's Universal Spanish-English Dictionary, the Graphical law and A Dictionary of Geography of Oxford University Press, viXra: 2103.0175[Condensed Matter].
20. Biswas Anindya Kumar, A Dictionary of Modern Italian, the Graphical law and Dictionary of Law and Administration, 2000, National Law Development Foundation", viXra: 2107.0171[Condensed Matter].
21. Biswas Anindya Kumar, Langenscheidt's German-English English-German Dictionary and the Graphical law, viXra: 2107.0179[Linguistics].
22. Biswas Anindya Kumar, Essential Dutch dictionary by G. Quist and D. Strik, the Graphical law Classification, viXra: 2108.0040[Linguistics].
23. Biswas Anindya Kumar, Swahili, a lingua franca, Swahili-English Dictionary by C. W. Rechenbach and the Graphical law, viXra: 2108.0101[Linguistics].
24. Biswas Anindya Kumar, The French, Larousse Dictionnaire De Poche and the Graphical law, viXra: 2109.0080[Linguistics].
25. Biswas Anindya Kumar, An Arabic dictionary: "al-Mujam al-w'\{a}fi" or, "adhunik arabi-bangla abhidhan" and the Onsager's solution, viXra: 2109.0119[Condensed Matter].
26. Biswas Anindya Kumar, Langenscheidt Taschenwörterbuch Deutsch-Englisch/Englisch-Deutsch, Völlige Neubearbeitung and the Graphical law", viXra: 2109.0141[Linguistics].
27. Biswas Anindya Kumar, Lyngkhai Bawansuk, The Graphical law behind the NTC's Hebrew and English Dictionary by Arie Comey and Naomi Tsur, viXra: 2109.0164[Linguistics].
28. Biswas Anindya Kumar, Oxford Dictionary of Media and Communication and the Graphical law, viXra: 2109.0202[Social Science].
29. Biswas Anindya Kumar, Oxford Concise Dictionary of Mathematics, Penguin Dictionary of Mathematics and the Graphical law, viXra: 2112.0054[Social Science].
30. Biswas Anindya Kumar, An Arabic dictionary: "al-Mujam al-w'\{a}fi" or, "adhunik arabi-bangla abhidhan" and the Onsager's solution Second part, viXra: 2201.0021[Condensed Matter].
31. Biswas Anindya Kumar, The Penguin Dictionary of Sociology and the Graphical law, viXra: 2201.0046[Social Science].
32. Biswas Anindya Kumar, The Concise Oxford Dictionary of Politics and the Graphical law, viXra: 2201.0069[Social Science].
33. Biswas Anindya Kumar, A Dictionary of Critical Theory by Ian Buchanan and the Graphical law, viXra: 2201.0136[Social Science].
34. Biswas Anindya Kumar, The Penguin Dictionary of Economics and the Graphical law, viXra: 2201.0169[Economics and Finance].
35. Biswas Anindya Kumar, The Concise Gojri-English Dictionary by Dr. Rafeeq Anjum and the Graphical law, viXra: 2201.0205[Linguistics].
36. Biswas Anindya Kumar, A Dictionary of the Kachin Language by Rev.o.Hanson and the Graphical law, viXra: 2202.0030[Linguistics].
37. Biswas Anindya Kumar, A Dictionary of World History by Edmund Wright and the Graphical law, viXra: 2202.0130[History and Philosophy of Physics].
38. Biswas Anindya Kumar, Ekagi-Dutch-English-Indonesian Dictionary by J. Steltenpool and the Onsager's solution, viXra: 2202.0157[Condensed Matter].
39. Biswas Anindya Kumar, A Dictionary of Plant Sciences by Michael Allaby and the Graphical law, viXra: 2203.0011[Mind Science].
40. Biswas Anindya Kumar, Along the side of the Onsager's solution, the Ekagi language, viXra: 2205.0065[Condensed Matter].
41. Biswas Anindya Kumar, Along the side of the Onsager's solution, the Ekagi language-Part Three, viXra: 2205.0137[Condensed Matter].
42. Biswas Anindya Kumar, Oxford Dictionary of Biology by Robert S. Hine and the Graphical law, viXra: 2207.0089[Physics of Biology].
43. Biswas Anindya Kumar, A Dictionary of the Mikir Language by G. D. Walker and the Graphical law, viXra: 2207.0165[Linguistics].
44. Biswas Anindya Kumar, A Dictionary of Zoology by Michael Allaby and the Graphical law, viXra: 2208.0075[Physics of Biology].
45. Biswas Anindya Kumar, Dictionary of all Scriptures and Myths by G. A. Gaskell and the Graphical law, viXra: 2208.0093[Religion and Spiritualism].
46. Biswas Anindya Kumar, Dictionary of Culinary Terms by Philippe Pilibossian and the Graphical law, viXra: 2211.0061[Social Sciences].
47. Biswas Anindya Kumar, A Greek and English Lexicon by H.G. Liddle *et al* simplified by Didier Fontaine and the Graphical law, viXra: 2211.0087[Linguistics].
48. Biswas Anindya Kumar, Learner's Mongol-English Dictionary and the Graphical law, viXra: 2211.0101[Linguistics].

49. Biswas Anindya Kumar, Complete Bulgarian-English Dictionary and the Graphical law, viXra: 2212.0009[Linguistics].
50. Biswas Anindya Kumar, A Dictionary of Sindhi Literature by Dr. Motilal Jotwani and the Graphical Law, viXra: 2212.0015[Social Sciences].
51. Biswas Anindya Kumar, Penguin Dictionary of Physics, the Fourth Edition, by John Cullerne, and the Graphical law, viXra: 2212.0072[History and Philosophy of Physics].
52. Biswas Anindya Kumar, Oxford Dictionary of Chemistry, the seventh edition and the Graphical Law, viXra: 2212.0113[Chemistry].
53. Biswas Anindya Kumar, A Burmese-English Dictionary, Part I-Part V, by J. A. Stewart and C. W. Dunn *et al*, head words and the Graphical Law, viXra: 2212.0127[Linguistics].
54. Biswas Anindya Kumar, The Graphical Law behind the head words of Dictionary Kannada and English written by W. Reeve, revised, corrected and enlarged by Daniel Sanderson, viXra: 2212.0185[Linguistics].
55. Biswas Anindya Kumar, Sanchayita and the Graphical Law, viXra: 2301.0075[Social Science].
56. Biswas Anindya Kumar, Samsad Bangla Abhidan and The Graphical Law, viXra: 2302.0026[Linguistics].
57. Biswas Anindya Kumar, Bangiya Shabdakosh and The Graphical Law, viXra: 2302.0060[Linguistics].
58. Biswas Anindya Kumar, Samsad Bengali-English Dictionary and The Graphical Law, viXra: 2304.0047[Linguistics].
59. Biswas Anindya Kumar, Rudyard Kipling's Verse and the Graphical Law, viXra: 2304.0207[Social Science].
60. Biswas Anindya Kumar, W. B. Yeats, The Poems and the Graphical Law, viXra: 2305.0008[Social Science].
61. Biswas Anindya Kumar, The Penguin Encyclopedia of Places by W. G. Moore and the Graphical Law, viXra: 2305.0147[Archaeology].
62. Biswas Anindya Kumar, The Poems of Tennyson and the Graphical Law, viXra: 2305.0157[Social Science].
63. Biswas Anindya Kumar, Khasi-Jaintia Jajids(Surnames) and the Graphical law, viXra:2307.0135[Social Science].
64. Biswas Anindya Kumar, Age, Amplitude of accommodation and the Graphical law, viXra:2311.0110[Physics of Biology].
65. Biswas Anindya Kumar, Dictionary of Ayurveda by Dr. Ravindra Sharma and the Graphical law, viXra:2401.0030[General Science and Philosophy].
66. Biswas Anindya Kumar, The Practical Sanskrit-English Dictionary by Vaman Shivram Apte and The Graphical Law, viXra:2402.0041[Linguistics].
67. Biswas Anindya Kumar, The Langenscheidt's Pocket Russian Dictionary and The Graphical Law, viXra:2402.0049[Linguistics].
68. Biswas Anindya Kumar, The Scholar Dictionary Portuguese and The Graphical Law, viXra:2402.0044[Linguistics].
69. Biswas Anindya Kumar, The Langenscheidt's Pocket Japanese Dictionary and the Onsager's solution, viXra:2402.0052[Condensed Matter].
70. Biswas Anindya Kumar, Langenscheidt's Pocket Chinese Dictionary and The Graphical Law, viXra:2403.0066[Linguistics].
71. Biswas Anindya Kumar, Oxford Hindi-English Dictionary and The Graphical Law, viXra:2403.0129[Linguistics].
72. Biswas Anindya Kumar, Concise Urdu to English Dictionary and The Graphical Law, viXra:2404.0006[Linguistics].
73. Biswas Anindya Kumar, The Standard Urdu-English Dictionary by Abdul Haq and The Graphical Law", viXra:2404.0034[Linguistics].
74. Biswas Anindya Kumar, The Urdu-Hindi Shabdakosh by Muhammad Sajjad Osmani, Sudhindra Kumar and The Graphical Law, viXra:2404.0114[Linguistics].
75. Biswas Anindya Kumar, The Hadronic Resonance Masses and The Graphical Law, viXra:2404.0141[High Energy Particle Physics].
76. Biswas Anindya Kumar, A Dictionary of British Surnames by P. H. Reaney and the Graphical Law, viXra:2408.0099[Social Science].
77. Biswas Anindya Kumar, Dictionary of Sports by Dr. S.K. Srivastava and Ms. Tanvangi Singh and the Graphical Law, viXra:2409.0014[Social Science].
78. Biswas Anindya Kumar, Dictionary of American Family Names by Elsdon C. Smith and the Graphical Law, viXra:2409.0049[Social Science].
79. Biswas Anindya Kumar, Political Map of Northeast India and the Graphical Law, viXra:2409.0092[Social Science].
80. Biswas Anindya Kumar, Dictionary of Computers Edited by Pankaj Dhaka and the Graphical Law, viXra:2409.0132[General Science and Philosophy].
81. Biswas Anindya Kumar, Swedish Dictionary and the Graphical Law, viXra:2409.0165[Linguistics].
82. Biswas Anindya Kumar, Tourist Guide and Map, Meghalaya and the Graphical Law, viXra:2410.0002[Social Science].
83. Biswas Anindya Kumar, Tourist Guide and Map, Aizawl and the Graphical Law, *IJASR*: 2024; 3(5):44-52.
84. Biswas Anindya Kumar, The Thai-English Student's Dictionary compiled by Mary R. Haas and the Graphical Law, viXra:2410.0093[Linguistics].
85. Biswas Anindya Kumar, Santali and The Graphical Law, viXra:2411.0151[Linguistics].
86. Biswas Anindya Kumar, The Oxford Dictionary of English Christian Names by E. G. Withycombe and the Graphical Law, viXra:2412.0008[Social Science].
87. Biswas Anindya Kumar, Pali-English Dictionary by T. W. Rhys Davids and William Stede and The Graphical Law, viXra:2412.0082[Linguistics].
88. Biswas Anindya Kumar, The Penguin Dictionary of Archaeology by Warwick Bray and David Trump and the Graphical Law, viXra:2412.0173[Archaeology].

89. Biswas Anindya Kumar, Cassell's New Latin Dictionary and The Graphical Law, viXra:2412.0183[Social Science].
90. Biswas Anindya Kumar, The Oxford Spanish Dictionary and The Graphical Law, viXra:2501.0122[Linguistics].
91. Biswas Anindya Kumar, Norwegian Dictionaries by H. Scavenius and The Graphical Law, viXra:2502.0026[Linguistics].
92. Biswas Anindya Kumar, Lushai Dictionary and The Graphical Law, viXra:2502.0073[Linguistics].
93. Biswas Anindya Kumar, Compact Oxford Italian Dictionary and The Graphical Law, viXra:2502.0197[Linguistics].
94. Biswas Anindya Kumar, A New Dictionary of the Portugese and English Languages enriched by H. Michaelis and The Graphical Law, viXra:2503.0102[Linguistics].
95. Biswas Anindya Kumar, Dictionary of Early English by Joseph T. Shipley and The Graphical Law, viXra:2503.0109[Linguistics].
96. Biswas Anindya Kumar, A Dictionary of the Dano-Norwegian and English Languages by A. Larsen and The Graphical Law, viXra:2504.0015[Linguistics].
97. Biswas Anindya Kumar, The Onsager solution, The Hawaiian Language, viXra:2504.0120[Condensed Matter].
98. Biswas Anindya Kumar, An Indonesian-English Dictionary by John M. Echols and Hassan Shadily and The Graphical Law, viXra:2504.0122[Linguistics].
99. Biswas Anindya Kumar, A Malay-English Dictionary (Romanised) by R. J. Wilkinson and The Graphical Law, viXra:2505.0093[Linguistics].
100. Biswas Anindya Kumar, The Vietnamese-English Dictionary by Mrs. Le Van Hung and Dr. Le Van Hung and The Graphical Law, viXra:2505.0103[Linguistics].
101. Biswas Anindya Kumar, Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch and The Graphical Law, viXra:2505.0113[Linguistics].
102. Biswas Anindya Kumar, Dictionary of The Hausa Language by R.C. Abraham and The Graphical Law, viXra:2505.0112[Linguistics].
103. Biswas Anindya Kumar, A Concise Dictionary of Slang and Unconventional English and The Graphical Law, viXra:2505.0187[Social Science].
104. Gun A. M., Gupta M. K. and Dasgupta B., Fundamentals of Statistics Vol 1, Chapter 12, eighth edition, 2012, The World Press Private Limited, Kolkata.
105. Ising E. 1925: Beitrag zur Theorie des Ferromagnetismus, Z. Physik 31:253-258.
106. Pathria R. K. 1993: Statistical Mechanics, Pergamon Press, 400-403. ISBN: 0 08 018994 6 Flexicover.
107. Kittel C. 1994: Introduction to Solid State Physics, Fifth edition, thirteenth Wiley Eastern Reprint, Wiley Eastern Limited, New Delhi, India: p438. ISBN: 0-85226-469-0.
108. Bragg W. L. and Williams E. J. 1934: The Effect of Thermal Agitation on Atomic Arrangement in Alloys, Proc. Roy. Soc. A, 145, p. 699. DOI: <https://doi.org/10.1098/rspa.1934.0132>.
109. Chaikin P. M. and Lubensky T. C. 1998: Principles of Condensed Matter Physics, p. 148, first edition, Cambridge University Press India Pvt. Ltd, New Delhi. ISBN 10: 81-7596-025-6 Paperback.
110. Huang K. 1987: Statistical Mechanics, second edition, John Wiley and Sons(Asia) Pte Ltd. ISBN: 9971-51-295-5
111. Borgnakke S. and Wylen V. 1998: Fundamentals of Thermodynamics, fifth edition, John Wiley and Sons Inc.: 206-207.