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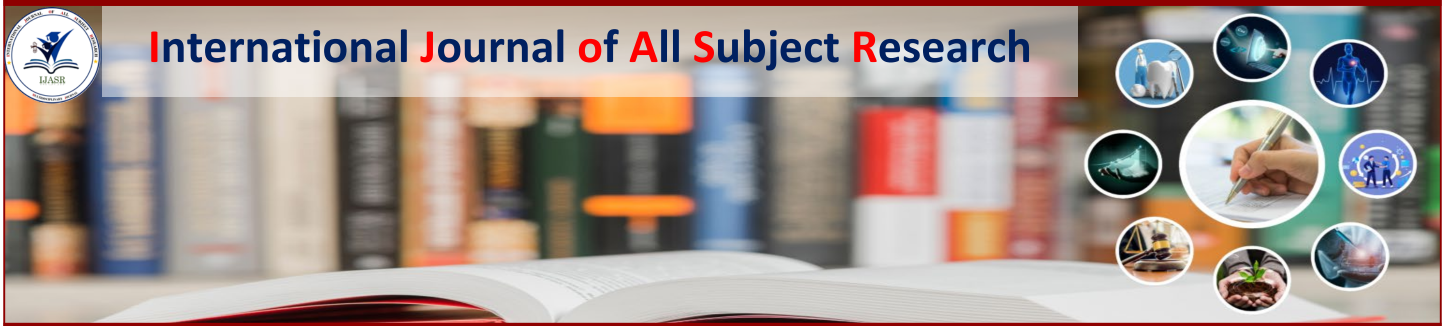

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A Study of an Identical-Strength Warm Standby System with Gamma Distribution and Imperfect Switching

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Abstract

In this paper, we consider a warm standby redundant system with an imperfect switching. The strengths of the components are assumed to follow identical Gamma distributions. A general expression for the reliability of the system R_n, is derived under the assumed stress–strength distribution. Numerical results are obtained and presented in tabular form to illustrate the reliability performance of the system for different parameter values of the Gamma stress–strength distributions.

Keywords: Stress-strength, gamma distribution, warm standby, imperfect switch.

1. Introduction

In a standby redundant system, when the active component fails, the next available standby component is immediately activated by a switching device known as a switch. In reliability analysis, the switch is generally assumed to be perfectly reliable. However, in practical situations, the switch may also be subject to failure. Therefore, while evaluating the reliability of a standby redundant system, it is essential to consider not only the failure mechanisms of the individual components but also the failure behaviour of the switching device. Depending on the operating condition of the standby components, a standby redundant system may be classified into three types: hot standby, cold standby, and warm standby systems. In a warm standby system, the redundant components remain in a partially energized or partially active state until they are required to replace a failed primary component. Since the standby components are partially energized during the standby period, they may also fail before being activated. However, their failure probability is generally lower than that of the active component. Thus, a realistic reliability analysis of a warm standby system should account for both the possible failures of the standby components and the imperfect operation of the switching mechanism.

2. Literature Survey

The stress–strength reliability model has been extensively studied by Kapur and Lamberson [5]. Sriwastav and Dutta [8] investigated imperfect switching in cold standby systems under the stress–strength model, considering both the switches and system components to follow similar distributions. Sriwastav [7] investigated warm standby systems with imperfect switching under the stress–strength model, considering dissimilar continuous distributions for the switches and components. Alidrisi [1] studied imperfect switching in dynamic warm standby systems using the time-to-failure (TTF) model. Dutta and Bhowal [2] considered imperfect switching with identical component strengths in a cold standby redundant system. Subsequently, Dutta and Bhowal [3] examined the reliability of a standby system in which the switches and components follow dissimilar continuous distributions. Sadeghi and Ronghanian [6] studied a warm standby repairable system including two dissimilar units including one repairman and imperfect switching mechanism. They assumed times to failure and times to repair of active and standby units as exponentially distributed. Gajjar and Patel [4] further studied the system reliability of a cold standby system with imperfect switching using a discrete stress–strength model. However, relatively little attention has been given to the reliability analysis of a warm standby system with imperfect switching when the component strengths are identical and follow similar continuous distributions.

The main objective of the present study is to derive the system reliability, R_n, of an n unit warm standby redundant system with imperfect switching, assuming identical component strengths following a Gamma distribution. The derived reliability expression is further evaluated numerically to examine the performance of the proposed system under different parameter values.

Section 3 presents methodologies where the mathematical formulation of the proposed model and the reliability expression for an n-unit warm standby system with an imperfect switching is derived. Section 4 examines the results

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of Gamma Stress-Strength for identical Strength and the corresponding reliability expression. Section 5 presents numerical results in tabular form to illustrate the variation in system reliability with respect to the parameters involved. Finally, section 6 and 7 represents conclusion and references.

3. Methodology

Let the strengths of the n components be identical and denoted by X. Let $Y_1, Y_2, \ldots, Y_n$ and $Z_2, Z_3, \ldots, Z_n$ be the stresses on the n^{th} component when it is active and it is standby respectively. Let U be the strength and V be the stress on the switch. The switch fails when $U \langle V$. We assume that X, Y_i, Z_j, U and V are all independent random variables.

The reliability R_n of the system is given by

$$R_n = R(1)+R(2)+R(3)+\ldots+R(n) \tag{2.1}$$

But now R(r), r=1,2,…,n is given as follows

$$R(1) = P[X \geq Y_1] \tag{2.2}$$

$$R(2) = P[X\langle Y_1, \{X \geq Z_2, (U \geq V and X \geq Y_2)\}] \tag{2.3}$$

$$R(3) = P[X\langle Y_1, \{X \geq Z_2, (U \geq V and X\langle Y_2) or X\langle Z_2\}\{X \geq Z_3, (U \geq V and X \geq Y_3)\}] \tag{2.4}$$

Then in general, we have

$$R(r) = P[X\langle Y_1, \{X \geq Z_2, (U \geq V and X\langle Y_2) or X\langle Z_2\}\{X \geq Z_3, (U \geq V and X\langle Y_3) or X\langle Z_3\}, \ldots, \{X \geq Z_{r-1}, (U \geq V and X\langle Y_{r-1}) or X\langle Z_{r-1}\}\{X \geq Z_r, (U \geq V and X \geq Y_r)\}] \tag{2.5}$$

Let f(x), $g_i(y)$, $w_j(z)$, h(u) and k(v), i=1,2,…,n; j=2,3,…,n be the p.d.f.'s of X, Y_i, Z_j, and V respectively. Since all the components and the switch work independently, we have

$$R(1) = \int_{-\infty}^{\infty} \overline{F}(y) g_1(y) dy \tag{2.6}$$

$$R(2) = \int_{-\infty}^{\infty} F(y) g_1(y)\, dy \int_{-\infty}^{\infty} \overline{F}(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} \overline{F}(y) g_2(y) dy \tag{2.7}$$

$$R(3) = \{\int_{-\infty}^{\infty} F(y) g_1(y) dy\}\{\int_{-\infty}^{\infty} \overline{F}(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} F(y) g_2(y) dy + \int_{-\infty}^{\infty} F(z) w_2(z) dz\}\{\int_{-\infty}^{\infty} \overline{F}(z) w_3(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} \overline{F}(y) g_3(y) dy\} \tag{2.8}$$

Then,

$$R(r) = \{\int_{-\infty}^{\infty} F(y) g_1(y) dy\}\{\int_{-\infty}^{\infty} \overline{F}(z) w_2(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} F(y) g_2(y) dy + \int_{-\infty}^{\infty} F(z) w_2(z) dz\}\{\int_{-\infty}^{\infty} \overline{F}(z) w_3(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} \overline{F}(y) g_3(y) dy + \int_{-\infty}^{\infty} F(z) w_3(z) dz\} \ldots \{\int_{-\infty}^{\infty} \overline{F}(z) w_r(z) dz \int_{-\infty}^{\infty} \overline{H}(v) k(v) dv \int_{-\infty}^{\infty} \overline{F}(y) g_r(y) dy\} \tag{2.9}$$

$where, r = 1,2,\ldots,n$

Here F(x) and H(u) are the c.d.f.'s of X and U respectively.

4. Results: Gamma Stress-Strength for Identical Strength

Let f(x), $g_i(y)$, $w_j(z)$, h(u) and k(v), i=1,2,…,n; j=2,3,…,n be all gamma densities with shape parameters $\theta, \alpha_i, \beta_j, \lambda and \mu$ respectively and scale parameters equal to unity.

$$f(x) = \begin{cases} \frac{1}{\Gamma\theta} e^{-x} x^{\theta-1}, x \geq 0, \theta \geq 1 \\ 0, otherwise \end{cases}$$

$$g_i(y) = \begin{cases} \frac{1}{\Gamma\alpha_i} e^{-y_i} {y_i}^{\alpha_i-1}, y_i \geq 0, \alpha_i \geq 1 \\ 0, otherwise \end{cases}$$

$$w_j(z) = \begin{cases} \frac{1}{\Gamma\beta_j} e^{-z_j} {z_j}^{\beta_j-1}, z_j \geq 0, \beta_j \geq 1 \\ 0, otherwise \end{cases}$$

$$h(u) = \begin{cases} \frac{1}{\Gamma\lambda} e^{-u_i} {u_i}^{\lambda-1}, u_i \geq 0, \lambda \geq 1 \\ 0, otherwise \end{cases}$$

$$k(v) = \begin{cases} \frac{1}{\Gamma\mu} e^{-v_i} {v_i}^{\mu-1}, v_i \geq 0, \mu \geq 1 \\ 0, otherwise \end{cases}$$

From (2.6), (2.7) and (2.8) we get

$$R(1) = R(\theta, \alpha_1)$$

$$R(2) = \overline{R}(\theta, \alpha_1) R(\theta, \beta_2) R(\lambda, \mu) R(\theta, \alpha_2)$$

$$R(3) = \overline{R}(\theta, \alpha_1)\{R(\theta, \beta_2) R(\lambda, \mu) \overline{R}(\theta, \alpha_2) + \overline{R}(\theta, \beta_2)\}\{R(\theta, \beta_3) R(\lambda, \mu) R(\theta, \alpha_3)\}$$

where,

$R(\theta, \alpha_r) = \sum_{i=0}^{\theta-1} \frac{\Gamma(\theta+\alpha_r-i-1)}{\Gamma\alpha_r(\theta-i-1)!2^{\theta+\alpha_r-i-1}}, r = 1,2,\ldots,n,$

$$\overline{R}(\theta, \alpha_r) = 1 - R(\theta, \alpha_r)$$

$$R(\theta, \beta_j) = \sum_{i=0}^{\theta-1} \frac{\Gamma(\theta + \beta_j - i - 1)}{\Gamma\beta_j(\theta - i - 1)!\,2^{\theta+\beta_j-i-1}}, j = 2,3,\ldots,n$$

$$\overline{R}(\theta, \beta_j) = 1 - R(\theta, \beta_j)$$

$$R(\lambda, \mu) = \sum_{i=o}^{\lambda-1} \frac{\Gamma(\lambda + \mu - i - 1)}{\Gamma\mu(\lambda - i - 1)!\,2^{\lambda+\mu-i-1}}$$

Then the system reliability R_3 from (2.1) is given by

$$R_3 = R(1) + R(2) + R(3).$$

5. Numerical Evaluation

For a few values of the parameters involved in the expressions of R(r), r=1,2,3 we evaluate R(1), R(2), R(3) and R_3 for different distributions from their expressions obtained in the last section.

Table 1: Gamma Stress-Strength: For Identical Strength

θ	α_1	α_2	α_3	β_2	β_3	λ	μ	R(1)	R(2)	R(3)	R_3
1	.5	.5	.5	.5	.5	1	1	.7071	.0732	.0290	.8094
	1	1	1	1	1	1	1	.5000	.0625	.0391	.2711
	2	2	2	2	2	1	2	.2500	.0117	.0093	.1282
	3	3	3	3	3	1	3	.1250	.0017	.0015	.6016
2	1	1	1	1	1	2	1	.7500	.1055	.0412	.8967
	2	2	2	2	2	2	2	.5000	.0625	.0391	.6016
	3	3	3	3	3	2	3	.3125	.0210	.0158	.3493

6. Conclusion

From the Table: 5.1, it is clear that for some values of the parameters if θ increases then the system reliability increase. i.e., if $\theta = 1$, $R_3 = .6016$ and if $\theta = 2$, $R_3 = .3493$. But if $\alpha_1$increases reliability $R(1)$ decreases. For example, $\alpha_1 = 1 (\theta = 2, \alpha_2 = \alpha_3 = \beta_2 = \beta_3 = \mu = 1, \lambda = 2), R(1) = .7500$ *and* $\alpha_1 = 2, R(1) = .5000$. If α_2 and β_2 increases then marginal reliability $R(2)$ decreases. Similar conclusion can be drawn for α_3 and β_3.

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